**Lab 07-Set Operations**

**Objective**

Solving exercises from the textbook in chapter 2.1 & 2.2

**Current Lab Learning Outcomes (LLO)**

By completion of the lab, the students should be able to:

1. will understand set theory.

2. will be able to solve shorter/easier or longer / harder problems given in the textbook

**Lab Requirements**

Students allowed using their lecture notes in the lab in order to solve the exercises.

**Lab Assessment**

1- Divide students to groups and let them to solve the given example.

2- Discuss the answers with the groups and write on board the optimal solution.

**Lab Description**

1. List the members of these sets.

a) {x | x is a real number such that x2 = 1}

b) {x | x is a positive integer less than 12}

c) {x | x is the square of an integer and x < 100}

d) {x | x is an integer such that x2 = 2}

Solution



1. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first,or neither is a subset of the other.

**a)** the set of airline flights from NewYork to New Delhi, the set of nonstop airline flights from New York to New Delhi

**b)** the set of people who speak English, the set of people who speak Chinese

**c)** the set of flying squirrels, the set of living creatures that can fly

**a)** The second is a subset of the first, but the frist is not a subset of the second **b)** Neither is a subset of the other. **c)** The first is a subset of the second, but the second is not a subset of the first

1. For each of the following sets, determine whether 2 is an element of that set.

**a)** {*x* ∈ **R** | *x* is an integer greater than 1}

**b)** {*x* ∈ **R** | *x* is the square of an integer}

**c)** {2*,*{2}} **d)** {{2}*,*{{2}}}**e)** {{2}*,*{2*,*{2}}} **f )** {{{2}}}

**a)** Yes **b)** No **c)** Yes **d)** No **e)** No **f)** No

1. Find the truth set of each of these predicates where the domain is the set of integers.

**a)** *P(x)*: *x*2 *<* 3 **b)** *Q(x)*: *x*2 *> x* **c)** *R(x)*: 2*x* + 1 = 0

**a)** {−1*,* 0*,* 1} **b) Z**−{0*,* 1} **c)** ∅

1. Let *A* be the set of students who live within one mile of school and let *B* be the set of students who walk to classes. Describe the students in each of these sets.

**a)** *A* ∩ *B* **b)** *A* ∪ *B* **c)** *A* − *B* **d)** *B* − *A*

**a)** The set of students who live within one mile of school and walk to classes **b)** The set of students who live within one mile of school or walk to classes (or do both) **c)** The set of students who live within one mile of school but do not walk to classes **d)** The set of students who walk to classes but live more than one mile away from school

1. Determine whether each of these statements is true or false.

**a)** 0 ∈ ∅ **b)** ∅ ∈ {0} **c)** {0} ⊂ ∅ **d)** ∅ ⊂ {0}

**e)** {0} ∈ {0} **f )** {0} ⊂ {0} **g)** {∅} ⊆ {∅}

Solution



1. What is the cardinality of each of these sets?
   1. {*a*} **b)** {{*a*}} **c)** {*a,* {*a*}} **d)** {*a,* {*a*}*,* {*a,* {*a*}}}

Solution: **a)** 1 **b)** 1 **c)** 2 **d)** 3

1. Find the power set of the sets where *a* and *b* are distinct elements
   1. {*a, b*} b) {∅*,* {∅}}

Solution: a) {∅*,* {*a*}*,* {*b*}*,* {*a, b*}} b) {∅*,* {∅}*,* {{∅}}*,* {∅*,* {∅}}}

1. How many elements does each of these sets have where *a* and *b* are distinct elements?

**a)** *P(*{*a, b,* {*a, b*}}*) 8*

**b)** *P(*{∅*, a,* {*a*}*,* {{*a*}}}*) 16*

**c)** *P(P(*∅*)) 2*

1. Let *A* = {*a, b, c, d*} and *B* = {*y, z*}. Find
   1. *A* × *B*. **b)** *B* × *A*.

Solution: **a)** {*(a, y)*, *(b, y)*,*(c, y)*, *(d, y)*, *(a, z)*, *(b, z)*, *(c, z)*, *(d, z)*} **b)** {*(y, a)*, *(y, b)*,*(y, c)*, *(y, d)*, *(z, a)*, *(z, b)*, *(z, c)*, *(z, d)*}

1. What is the Cartesian product *A* × *B* × *C*, where *A* is the set of all airlines and *B* and *C* are both the set of all cities in the United States? Give an example of how this Cartesian product can be used.

Solution: The set of triples *(a, b, c)*, where *a* is an airline and *b* and *c* are cities. A useful subset of this set is the set of triples *(a, b, c)* for which *a* flies between *b* and *c*.

1. Let *A* = {*a, b, c, d, e*} and *B* = {*a, b, c, d, e, f, g, h*}.Find
   1. **a)** *A* ∪ *B*. **b)** *A* ∩ *B*. **c)** *A* − *B*. **d)** *B* − *A*.

Solution: a) {a,b,c,d,e,f,g,h} b) {*a, b, c, d, e*} c) {} d) {f,g,h}

1. Let *A* = {0*,* 2*,* 4*,* 6*,* 8*,* 10}, *B* = {0*,* 1*,* 2*,* 3*,* 4*,* 5*,* 6}, Find |A∪B|. find the minimum number of elements in A∪B and maximum number of elements in A∪B

|A∪B|=|A|+|B|−|A∩B|

=6+7-4

9

A∪B is maximum or minimum according as |A∩B| is max or min.

case 1: |A∩B| is min ie |A∩B|=0

A∪B|=6+7-0

=13 so maximum number of elements in A∪B|is 13.

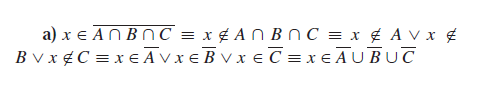
case 2: |A∩B| is max ie 4 , so and min number of elements in A∪B is 9. the generalized solution is below

The Min number of elements in (A U B): we should assume that the intersection is the maximum which is

      |A**∩**B|=|A| when A⊆B             or              |A**∩**B|=|B| when B⊆A  . so

           |A U B| = |B|                       or                       |A U B| =|A|

1. Show that if *A*, *B*, and *C* are sets, then =*A* ∪ *B* ∪ *C*
   1. by showing each side is a subset of the other side.

Solution: 

* 1. using a membership table.



1. Let *A*, *B*, and *C* be sets. Show that

**a)** *(A* ∪ *B)* ⊆ *(A* ∪ *B* ∪ *C)*.

**b)** *(B* − *A)* ∪ *(C* − *A)* = *(B* ∪ *C)* − *A*.

a)Solution: Let x be arbitrary.

x ∈ A ∪ B = (x ∈ A ∪ B) ∨ (x ∈ C)

= x ∈ (A ∪ B) ∪ C [Definition of ∪nion] Thus, since x ∈ A ∪ B → x ∈ (A ∪ B) ∪ C, it follows that

A ∪ B ⊆ A ∪ B ∪ C, by definition of subset

b)Proof. (B − A) ∪ (C − A)

= (B ∩ Ā ) ∪ (C ∩ Ā ) set difference

= (B ∪ C) ∩ Ā distibutive

= (B ∪ C) − A set difference

or

Let x ∈ (B − A) ∪ (C − A).

Then x ∈ (B − A) or x ∈ (C − A). If x ∈ (B − A), then x ∈ B and x  A by definition of set difference

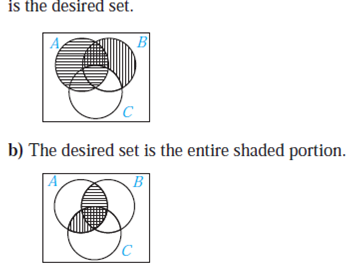
. If x ∈ (C − A), then x ∈ C and x A by definition of set difference.

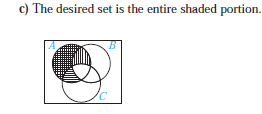
Thus, x ∈ B or x ∈ C and x  A. By definition of union, x ∈ (B∪C). By definition of set difference, if x ∈ (B ∪ C) and x A then x ∈ (B ∪ C) − A.

1. Draw the Venn diagrams for each of these combinations of the sets *A*, *B*, and *C*.
   1. **a)** *A* ∩ *(B* − *C)* **b)** *(A* ∩ *B)* ∪ *(A* ∩ *C) *

*Solution:*



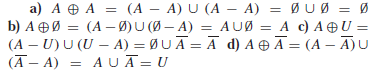




1. Show that if A is a subset of a universal set U, then

**a)** *A* ⊕ *A* = ∅. **b)** *A*⊕∅ = *A*. **c)** *A* ⊕ *U* = **. **d)** **= *U*.

Solution



1. Draw a Venn diagram that shows the following sample space and events:

*S*: all the integers from 1 to 30

*P*: prime numbers

*M*: multiples of 3

*F*: factors of 30

**Solution**

**Write down the sample space and event sets**

The sample space contains all the positive integers up to 30.

*S*={1;2;3;…;30}

The prime numbers between 1 and 30 are

*P*={2;3;5;7;11;13;17;19;23;29}

The multiples of 3 between 1 and 30 are

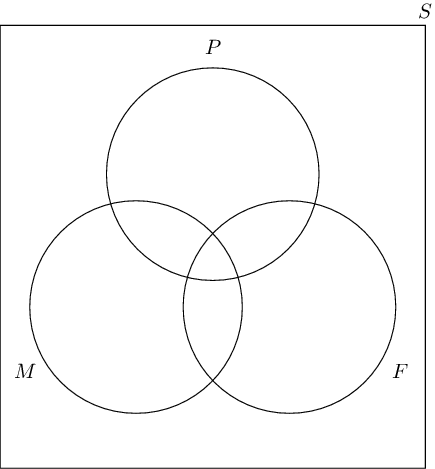
*M*={3;6;9;12;15;18;21;24;27;30}

The factors of 30 are

*F*={1;2;3;5;6;10;15;30}

**Draw the outline of the Venn diagram**

There are 3 events, namely *P*, *M* and *F*, and the sample space, *S*. Put this information on a Venn diagram:



**Place the outcomes in the appropriate event sets**

