Faculty of Computing & Information Technology
Computer Science Department

Assignment #2 - CPCS 222 (Winter 2022-23)
(Boy's campus)

Due: Wednesday (01/02/2023)
Student Name :____________________________________Student ID:_____________
Section : _______________

1. Sets

a) Let A = {1, a, b}, B = {p, 2} and C = {r, {∅}}. Find

i. R = C x B x A =

{ (r,p,1) , (r,p,a) , (r,p,b) , (r,2,1), (r,2,a), (r,2,b) , ({∅},p,1) , ({∅},p,a) , ({∅},p,b)
({∅},2,1) , ({∅},2,a) , ({∅},2,b) }

ii. |R| + |A x B| + |P(A)| =

|R| = 12
|A x B| = 3 x 2 = 6
|P(A)| = $2^n = 2^3 = 8$
12 + 6 + 8 = 26

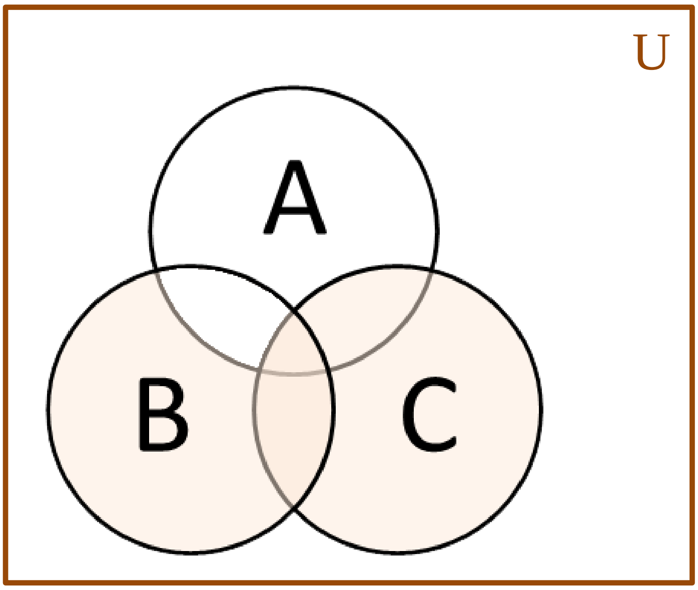
b) Let A = {7, 3, 7, 1, 4, 6, 2, 9} and B = {1, 2, 5, 4, 3, 3} and
C = {x | 2 ≤ x ≤ 5}. U = {x | 0 ≤ x ≤ 9} Find

i. $(A \cup \overline{B}) \cap (A - C)$

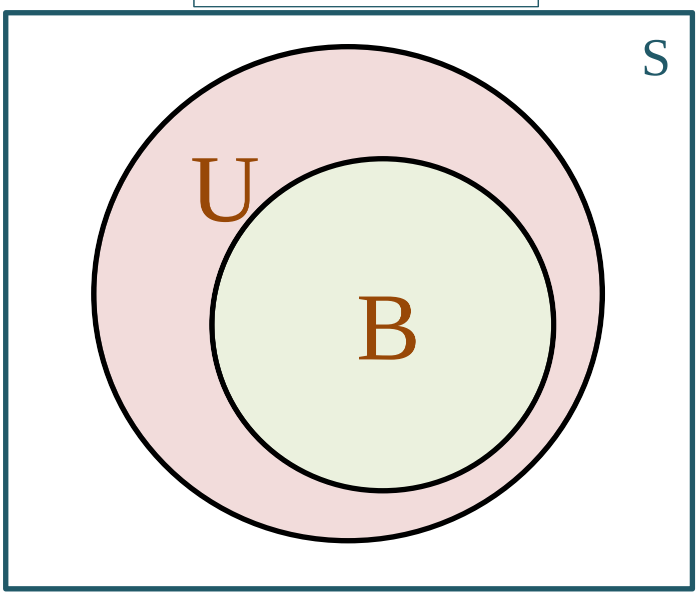
A = {7, 3, 7, 1, 4, 6, 2, 9} | $\overline{A}$ = {0,5,8}
B = {1, 2, 5, 4, 3, 3} | $\overline{B}$ = {0,6,7,8,9}
A∪$\overline{B}$= {0,1,2,3,4,6,7,8,9}
$A - C$ = {1,6,7,9}
$(A \cup \overline{B}) \cap (A - C)$ = {1,6,7,9}

ii. Draw the Venn diagrams of $(B \cap \overline{A}) \cup C$ and $B \subseteq U$

Venn diagram of $(B \cap \overline{A}) \cup C$



Venn diagram of $B \subseteq U$



2. Summations

a) $\sum_{x=1}^{15}(3x^2+4x)$

$$\sum_{x=m}^{n} f(x)+g(x)=\sum_{x=m}^{n} f(x)+\sum_{x=m}^{n} g(x) \;=\sum_{x=1}^{15} 3(x)^2+\sum_{x=1}^{15} 4(x)$$

$$=\;3\sum_{x=1}^{15}(x)^2+4\sum_{x=1}^{15}(x)=3\left\{\frac{15(15+1)(30+1)}{6}\right\}$$

$$+\,4\left\{\frac{15(15+1)}{2}\right\}=3(1240)+4(120)=3720+480$$

$$=4200$$

b) $\sum_{i=n-6}^{n}\sum_{j=1}^{n}(2i+6j^2)$

$$\sum_{j=m}^{n} f(j)+g(j)=\sum_{j=1}^{n} 2(i)+\sum_{j=1}^{n} 6(j)^2 = 2(i)\sum_{j=1}^{n} 1+6\sum_{j=1}^{n}(j)^2$$

$$=2in+6\left(\frac{n(n+1)(2n+1)}{6}\right)=2in+\big(n(n+1)(2n+1)\big)$$

$$=2in+(2n^3+2n^2+n^2+n)=$$

تقدر تبدأ بالسطر البني ذا لكن أنا اخترت اللي قبله مدري ليش بس النفس وما تهوى.
إذا بتبدأ فيه اكتبه وإذا بتسوي مثلي لا تكتبه

$$\sum_{i=n-6}^{n} 2in+\big(n(n+1)(2n+1)\big)=\sum_{i=n-6}^{n} 2in+\sum_{i=n-6}^{n}\big(n(n+1)(2n+1)\big)=$$

$$(2n)\sum_{i=n-6}^{n} i+\big(n(n+1)(2n+1)\big)\sum_{i=n-6}^{n} 1 \rightarrow \sum_{i=m}^{n} f(i)=\sum_{i=1}^{n} f(i)-\sum_{i=1}^{m-1} f(i)\rightarrow$$

$$(2n)\left(\sum_{i=1}^{n} i-\sum_{i=1}^{n-7} i\right)+\big(n(n+1)(2n+1)\big)\left(\sum_{i=1}^{n} 1-\sum_{i=1}^{n-7} 1\right)=$$

$$(2n)\left(\sum_{i=1}^{n} i-\sum_{i=1}^{n-7} i\right)+\big(n(n+1)(2n+1)\big)\left(\sum_{i=1}^{n} 1-\sum_{i=1}^{n-7} 1\right)=$$

$$(2n)\left(\left(\frac{n(n+1)}{2}\right)-\left(\frac{n-7\big((n-7)+1\big)}{2}\right)\right)+\big(n(n+1)(2n+1)\big)\big(n-(n-7)\big)=$$

$$(2n)\left(\left(\frac{n(n+1)}{2}\right)-\left(\frac{n-7\big((n-6)\big)}{2}\right)\right)+\big(n(n+1)(2n+1)\big)\big(n+(-n+7)\big)=$$

$$(2n)\left(\left(\frac{n(n+1)}{2}\right)-\left(\frac{n-7\big((n-6)\big)}{2}\right)\right)+\big(n(n+1)(2n+1)\big)(+7)=$$

$$(2n)\left(\left(\frac{(n^2+n)}{2}\right)-\left(\frac{(n^2-13n+42)}{2}\right)\right)+(2n^3+2n^2+n^2+n)(+7)=$$

$$(2n)\left(\left(\frac{(n^2+n)-(n^2-13n+42)}{2}\right)\right)+(14n^3+14n^2+7n^2+7n)=$$

$$(2n)\left(\left(\frac{(n^2+n)+(-n^2+13n-42)}{2}\right)\right)+(14n^3+14n^2+7n^2+7n)=$$

$$(2n)\left(\left(\frac{(14n-42)}{2}\right)\right)+(14n^3+21n^2+7n)=$$

$$(2n)\big((7n-21)\big)+(14n^3+21n^2+7n)=$$

$$(14n^2-42n)+(14n^3+21n^2+7n)=\ (14n^3+35n^2+-35n)$$

3. Sequences – Write the 10th term of the following sequences

a) 2.4, 3.6, 4.8, 6.0, 7.2, …… $\boxed{13.2}$

b) $\frac{1}{16}j^{-4}$, $-\frac{1}{8}j^{-3}$, $\frac{1}{4}j^{-2}$, $-\frac{1}{2}j^{-1}$, 1, …… $\boxed{-32j^5}$

4. Functions

a) Determine whether the function $f(x) = -2x + 5$ is a bijection from R to R $\boxed{\text{YES}}$

b) $\lfloor(\lceil \pi \rceil + \lfloor (1.4)^2 \rfloor)\rfloor =$ $\boxed{\lfloor\lceil 3.14\rceil + \lfloor 1.96\rfloor\rfloor = \lfloor 4 + 1\rfloor = \lfloor 5\rfloor = 5}$

c) Write the property of inverse function.

Bijection , denoted f^{-1} , defined as
$$f^{-1}(y) = x \text{ iff } f(x) = y$$

d) If $f(x) = -2x + 5$ and $g(x) = 3x^2 - 4$, find

i) $f \circ g(x) =$
$$-2(3x^2 - 4) + 5$$
$$-6x^2 + 8 + 5$$
$$\mathbf{-6x^2 + 13}$$

ii) $g \circ f(-1) =$
$$3(-2x + 5)^2 - 4$$
$$3(4x^2 - 20x + 25) - 4$$
$$12x^2 - 60x + 75 - 4$$
$$12x^2 - 60x + 71$$
$$12(-1)^2 - 60(-1) + 71$$
$$\mathbf{143}$$

5. Relations

a) For the following relations on the set {1, 2, 3, 4} decide whether it is reflexive, irreflexive, symmetric, anti-symmetric, transitive and equivalence
Equivalence = reflexive & symmetric & transitive.

i. {(2,2), (1,2), (3,3), (3, 2), (4,4)} – not-reflexive, not-irreflexive, not-symmetric, anti-symmetric, transitive, not-equivalent.

ii. {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)} – reflexive, not-irreflexive, symmetric, not anti-symmetric, transitive, **equivalent**.

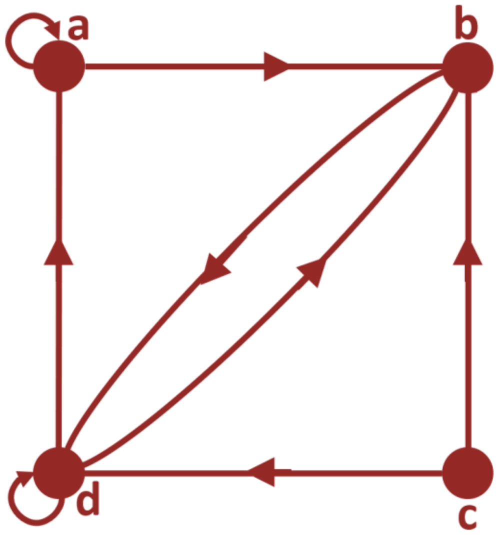
iii. {(1, 1), (1, 2), (2, 1), (2, 3), (3, 3), (3, 4), (4, 4)} – not-reflexive, not-irreflexive, not-symmetric, not anti-symmetric, not-transitive , not-equivalent.

b) Convert all the relations given above in 5a to matrix format.

Matrix size = |a|×|a| = 4×4.

$$M_{Ri} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, M_{Rii} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, M_{Riii} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Draw the directed graph that represents the relation {(a, a), (a, b), (b, d), (c, b), (c, d), (d, a), (d, b), (d, d)}.



Counting

a) What is the value of K and R after the following codes have been executed? (i)
K= n+2n+3n= 6n__

```
K:=0
for i1:=1 to n
        K:= K +1
for i2=1 to 2n
        K:= K +1
for i3=1 to 3n
        K:= K +1
```

ii)
```
R:=0
for i1:=1 to n
 for i2:=1 to 2n
 for i3:=1 to 3n
   R:= R +1
```

R= $3n \times 2n \times n = 6n^3$__

b) A student taking a mathematics exam is directed to answer any seven of 10 easy questions. How many ways are there to answer these questions?
any seven(7) means order is not important, so we will use combinations.
$$nCr = 10C7 = \frac{n!}{r!\,(n-r)!} = \frac{10!}{7!\,(10-7)!} = \frac{10!}{7! \times (3)!} = \frac{3{,}628{,}800}{30{,}240} = 120$$

c) How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries?
$(45C3) \times (57C4) \times \big(69C(12-3-4)\big) = (45C3) \times (57C4) \times (69C5) =$
$14{,}190 \times 395{,}010 \times 11{,}238{,}513 = 62{,}994{,}022{,}035{,}644{,}700 \approx$
$6.299402204 \times 10^{16}$

d) How many ways are there to arrange the letters a, b, c, and d such that a is not followed immediately by b?
4 letters can be arranged in 4! = 4 × 3 × 2 × 1= 24 ways.
Let ab as 1 letter, then we can arrange {ab , c , d} in 3! = 3 × 2 × 1 = 6 ways. So we can remove this 6 ways from the 24 ways.
4! – 3! = 24 – 6 = 18 ways.

e) How many strings of eight uppercase English letters are there.

i) If no letter can be repeated

English letters = 26.

$26\times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19 = 62{,}990{,}928{,}000 \approx 6.2990925 \times 10^{10}$

Or we can say 26P8 = $\frac{26!}{(26-8)!} = \frac{26!}{(18)!} = \frac{403291461126605635584000000}{6402373705728000} = 62{,}990{,}928{,}000 \approx 6.2990925 \times 10^{10}$

ii) The start and end can be repeated.

English letters = 26.

He want 1 fixed letter in start and end Like X------X.

$1\times 26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 1 = 165{,}765{,}600$

Or we can say $1 \times$ (26P6)$\times 1 = 1 \times \frac{26!}{(26-6)!} \times 1 = 20 \times \frac{26!}{(20)!} =$

$$1 \times \frac{403291461126605635584000000}{2432902008176640000} \times 1 = 1 \times 165{,}765{,}600 \times 1 = 165{,}765{,}600$$

f) A bowl contains 10 red balls and 10 blue balls. How many balls should we select at random without looking at them to definitely get

i) 5 balls of the same color?

Number of boxes k = 2

We want $\lceil N/k \rceil = 5 \rightarrow \lceil N/2 \rceil = 5$

minimum N = 9

ii) 4 blue balls?

Consider the “worst” case, Consider 10 red balls and 3 blue balls. We can’t take another ball without hitting 3 blue balls. Thus, the answer is 14

g) A computer company receives 350 applications from computer graduates for a job. Suppose that 220 of these people majored in CS, 147 majored in business, and 51 majored both in CS and in business. How many of these applicants will be rejected from these 350 applications?

The rejected applicants means that graduates neither CS nor business.

Let $|U|$= 350 , $|A|$=220 , $|B|$=147 , $|A \cap B| = 51$

We can find the rejected graduates by $|U| - |A \cup B| = |U| - (|A| + |B| - |A \cap B|) = 350 - (220 + 147 - 51) = 350 - 316 =$ 34 rejected applications.

h) What is the minimum number of students needed in a class to be sure that at least 5 born on the same month?

Number of boxes k = 12

We want $\lceil N/k \rceil = 5 \rightarrow \lceil N/12 \rceil = 5$

minimum N = 49

i) How many strings of nine English letters are there such that it contains at least three vowels (a, e, i, o, u)?
n=9.
U = $English\ letters^n = 26^9$.
We can say at least 3 = U – at most 2 .

$$at\ most\ 2 = \left((9C2) \times 5^2 \times 21^7\right) + \left((9C1) \times 5^1 \times 21^8\right) + \left((9C0) \times 5^0 \times 21^9\right) = (1{,}620{,}979{,}686{,}900) + (1{,}702{,}028{,}671{,}245) + (794{,}280{,}046{,}581) = 4{,}117{,}288{,}404{,}726$$

$$26^9 - 4{,}117{,}288{,}404{,}726 = 5{,}429{,}503{,}678{,}976 - 4{,}117{,}288{,}404{,}726 = 1{,}312{,}215{,}274{,}250 \approx 1.312215274 \times 10^{12}$$

Induction

Let P(n) be the statement that

$$\sum_{i=1}^{n}(i-2)(i+2)=\frac{n(n+1)(2n+1)}{6}-4n$$

a) Show that P(1) is true.

We want to confirm that the left side (LS) equals the right side (RS) when we plug in the smallest value (which in this case is 1).

LS: $\sum_{i=1}^{n}(i-2)(i+2)=\sum_{i=1}^{n}(1-2)(1+2)=\sum_{i=1}^{n}(-1)(3)=\ \mathbf{-3}$

RS: $\frac{n(n+1)(2n+1)}{6}-4n=\ \frac{1(1+1)(2(1)+1)}{6}-4(1)=\ \frac{1(2)(3)}{6}-4(1)=$

$$\frac{6}{6}-4=1-4=\mathbf{-3}$$

LS = RS , so P(1) is true.

b) What is the inductive hypothesis?

assume it is true for an arbitrary positive integer k when $k\geq 1$.

$$\sum_{i=1}^{k}(i-2)(i+2)=\frac{k(k+1)(2k+1)}{6}-4k$$

Assume P(k) is True when k≥1.

c) Prove the statement given above for all values of n using mathematical induction.

As we have done the Base Case and Inductive Hypothesis(I.H.). We have to do the inductive step.

Inductive Step : Show it is true for k+1 based on the truth of k

based on the I.H., prove P(k+1). We plug in a k+1 wherever we see k.

$$prove\ this: \sum_{i=1}^{k+1}(i-2)(i+2)$$
$$=\frac{(k+1)\big((k+1)+1\big)(2(k+1)+1)}{6}-4(k+1)$$

$$\sum_{i=1}^{k+1}f(i)=\sum_{i=1}^{k}f(i)+f(k+1)\rightarrow$$

$$\sum_{i=1}^{k+1}(i-2)(i+2)=\sum_{i=1}^{k}(i-2)(i+2)+((k+1)-2)((k+1)+2)$$

By induction hypothesis: $\sum_{i=1}^{k}(i-2)(i+2)=\frac{k(k+1)(2k+1)}{6}-4k$

$$\sum_{i=1}^{k+1}(i-2)(i+2)$$
$$=\left(\frac{k(k+1)(2k+1)}{6}-4k\right)+\big((k+1)-2\big)\big((k+1)+2\big)\rightarrow$$

$$\frac{(k+1)\big((k+1)+1\big)\big(2(k+1)+1\big)}{6}-4(k+1)$$
$$=\left(\frac{k(k+1)(2k+1)}{6}-4k\right)+\big((k+1)-2\big)\big((k+1)+2\big)\rightarrow$$

$$\left(\frac{k(k+1)(2k+1)}{6}-4k\right)+\big((k-1)\big)\big((k+3)\big)$$
$$=(6)\left(\left(\frac{k(k+1)(2k+1)}{6}-4k\right)+\big((k-1)\big)\big((k+3)\big)\right)=$$

$$\left((6)\left(\frac{k(k+1)(2k+1)}{6}-4k\right)+(6)\big((k-1)\big)\big((k+3)\big)\right)=$$

$$\left(\left((6)\frac{k(k+1)(2k+1)}{6}-4k(6)\right)+(6)\big((k-1)\big)\big((k+3)\big)\right)=$$

$$\Big(\big(k(k+1)(2k+1)-24k\big)+(6)\big((k-1)\big)\big((k+3)\big)\Big)=$$

$$\Big(\big(k(2k^2+k+2k+1)-24k\big)+(6)(k^2+3k-k-3)\Big)=$$

$$\Big(\big(k(2k^2+3k+1)-24k\big)+(6)(k^2+2k-3)\Big)=$$

$$\Big(\big((2k^3+3k^2+k)-24k\big)+(6k^2+12k-18)\Big)=$$

$$\Big((2k^3+9k^2-11k-18)\Big)\rightarrow(2k^3+9k^2-11k-18)$$
$$=\Big((2k^3+9k^2+13k+6)+(-24k-24)\Big)=$$

$$\Big((2k^3+9k^2+13k+6)-(24k+24)\Big)=$$

$$\Big((2k^3+3k^2+6k^2+9k+4k+6)-(24k+24)\Big)=$$

$$\Big((k^2+3k+2)(2k+3)-(24k+24)\Big)=$$
$$\Big(\big((k^2+2k+k+2)\big)(2k+3)-(24k+24)\Big)=$$
$$\Big(\big((k^2+2k+k+2)\big)\big((2k+2)+1\big)-(24k+24)\Big)=$$
$$\Big((k+1)(k+2)\big(2(k+1)+1\big)-(24k+24)\Big)=$$
$$\left(\frac{(k+1)(k+2)\big(2(k+1)+1\big)}{6}-(4k+4)\right)=$$
$$\left(\frac{(k+1)(k+2)\big(2(k+1)+1\big)}{6}-4(k+1)\right)$$

$$prove\ this: \sum_{i=1}^{k+1}(i-2)(i+2) = \frac{(k+1)\big((k+1)+1\big)(2(k+1)+1)}{6} - 4(k+1)$$

We just proved the Induction Step above. We proved that P(k+1) is true under the assumption that P(k) is true.

Since we completed all three steps, by Mathematical Induction, we know that P(n) is true for all positive integers n.

That is, we have proven, for all positive n, that

$$\sum_{i=1}^{n}(i-2)(i+2) = \frac{n(n+1)(2n+1)}{6} - 4n$$