**Lab 06 - Logic and Proof**

**Objective**

Solving exercises from the textbook in chapter 1.6-1.8

**Current Lab Learning Outcomes (LLO)**

By completion of the lab, the students should be able to:

1. Students will understand Methods of proof.

2. They will be able to solve shorter/easier or longer / harder problems given in the textbook.

**Lab Requirements**

Students allowed using their lecture notes in the lab in order to solve the exercises.

**Lab Assessment**

1- Divide students to groups and let them to solve the given example.

2- Discuss the answers with the groups and write on board the optimal solution.

**Lab Description**

In this lab, the following exercises are going to be solved and explained to them:

1. Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

Solution: The theorem is of the form p→q, where p: there are two numbers one rational and the other irrational, and q: their sum is irrational. Using proof by contradiction, assume that p is true and q is false. That is suppose we have two numbers x and y, where x is irrational and y is rational. Also suppose that their sum is rational. Then x+y=a/b , where b≠0. Since y is rational also then we have x+y = x+c/d =a/b, where b,d≠0. Then x=a/b-c/d=(ad-cb)/bd🡺 x is rational 🡺 p is false, Which contradicts our assumption. Then the theorem is correct using proof by contradiction.

1. Show that these statements about the integer *x* are equivalent:

(i) 3*x* + 2 is even, (*ii*) *x* + 5 is odd, (*iii*) *x*2 is even.

We prove that all these are equivalent to *x* being even. If *x* is even, then *x* = 2*k* for some integer *k*. Therefore3*x*+2 = 3 ·2*k*+2 = 6*k*+2 = 2*(*3*k*+1*)*, which is even, because it has been written in the form 2*t* , where *t* = 3*k* + 1.Similarly, *x* + 5 = 2*k* + 5 = 2*k* + 4 + 1 = 2*(k* + 2*)* + 1, so *x* + 5 is odd; and *x*2= *(*2*k)*2= 2*(*2*k*2*)*, so *x*2is even. For the converses, we will use a proof by contraposition. So assume that *x* is not even; thus *x* is odd andwe can write *x* = 2*k* + 1 for some integer *k*. Then 3*x*+2 = 3*(*2*k*+1*)*+2 = 6*k*+5 = 2*(*3*k*+2*)*+1, which is odd(i.e., not even), because it has been written in the form 2*t* +1, where *t* = 3*k* +2. Similarly, *x* +5 = 2*k* +1+5 = 2*(k* +3*)*,so *x* + 5 is even (i.e., not odd).

If x is odd , by the definition of an odd integer, it follows that *x* = 2*k* + 1, where *k* is some integer. We want to show that *x*2 is also odd. We can square both sides of the equation *x* = 2*k* + 1 to obtain a new equation that expresses *x*2. When we do this, we find that *x*2= *(*2*k* + 1*)*2= 4*k*2 + 4*k* + 1 = 2*(*2*k*2+ 2*k)* + 1. By the definition of an odd integer, we can conclude that *x*2 is an odd integer (it is one more than twice an integer). Consequently, we have proved that if *x* is an odd integer, then *x*2 is an odd integer.

1. Show that if *n* is an integer and *n*3 + 5 is odd, then *n* is even using

**a)** a proof by contraposition.

**b)** a proof by contradiction.

**a)** Assume that *n* is odd, so *n* = 2*k*+1 for some integer *k*. Then *n*3+5 = 2*(*4*k*3+6*k*2+3*k*+3*)*. Because

*n*3 + 5 is two times some integer, it is even.

**b)** Suppose that*n*3 + 5 is odd and *n* is odd. Because *n* is odd and the product

of two odd numbers is odd, it follows that *n*2 is odd and then that *n*3 is odd.

But then 5 = *(n*3 + 5*)* − *n*3 would have

to be even because it is the difference of two odd numbers. Therefore, the supposition that *n*3 +5 and *n* were both odd is wrong.

1. Prove the following statement by contradiction if n2 is odd, then n is odd.

The theorem is of the form p→q, where p: n2 is odd and q: n is odd. Using proof by contradiction, assume that p is true and q is false

which means n2 is odd and n is even

q is false means n is not odd which means n is even.

if n is even n=2k

n2 =(2k)2

=4k2

2(2k2) which is even, so p is false which contradicts out statement

the theorem is correct using proof by contradiction.

1. Show that at least ten of any 64 days chosen must fall on the same day of the week.

Solution: If we chose 9 or fewer days on each day of the week, this would account for at most 9 ·7 = 63 days. But we chose 64 days. This contradiction shows that at least 10 of the days we chose must be on the same day of the week.

1. Prove that If you are studying CPCS222 then you are human (domain is all people), what kind of proof did you use.

Solution: the proposition is true, because Q is true, using trivial proof.

1. Show that if x2≥0 then x2+2x+1≥0 (Domain is of all real numbers). what kind of proof did you use.

Solution: the proposition is true, because Q is true, using trivial proof

1. Show that if x2+1<0 then 2x2+5x+1= 0 (Domain is of all real numbers). what kind of proof did you use.

The proposition is vacuously true because x2<-1 is false for any real number. Vacuous proof.

1. Prove that if *x* and *y* are real numbers, then max*(x, y)* +min*(x, y)* = *x* + *y*. [*Hint:* Use a proof by cases, with the two cases corresponding to *x* ≥*y* and *x < y*, respectively.]

Solution: If *x* ≤*y*, then max*(x, y)* + min*(x, y)* = *y* + *x* = *x* + *y*. If *x* ≥*y*, then

max*(x, y)* + min*(x, y)* = *x* + *y*.Because these are the only two cases, the equality always holds.

1. Prove that there is no positive integer *n* such that *n*2+ *n*3= 100.

Solution: Notice that if n=5 then n3=125>100. So we should test the numbers n= 1, 2, 3 and 4.

12+13=2≠ 100

22+23=12 ≠ 100

32+33=36 ≠ 100

42+43=80 ≠ 100

Therefore there is no positive integer *n* such that *n*2+ *n*3= 100.

1. Show that the implication If x > 0, then x 2 + 5 > 0. Where x is any real number.

p:= x > 0 and q : x 2 + 5 > 0

q is always true for any value of x. so p →T is always true. trivial proof.

1. Prove that there are no solutions in integers *x* and *y* to th equation 2*x*2 + 5*y*2 = 14.

y can only be 0 or +,-1, because if y is |2| or greater, then 5y2 is already 20 or more---which is greater than 14.  
If y=0, then x2 = 7, and x =√7 which is is not an integer.  
If y= |1|, then x2 = 9/2, and x is not an integer.  
  
x can only be 0, |1|, or |2|. If x is |3| or greater, then 2x2 is already 18 or more.  
If x=0, y2 = 14/5 and y is not an integer.  
If x=|1|, y2 = 12/5 and y is not an integer.  
If x=|2|, y2 = 16/5 and y is not an integer.  
  
Therefore, there is no solution in x and y as integers.

1. prove that the following implication is true, where x is a real number and n is a constant.

what ever the value of x then x2 + 1 > 0 is always true . So without considering p, q is always true.

like P →T is always true. trivial proof.

therefore the implication is true using trivial proof.