Assignment #1 - CPCS 222 (Winter 2022-23)
(Boy's campus)

Due: Sunday (08/01/2023)
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Section: F1

1. Construct a truth table for $(a \to \neg b) \land (\boldsymbol{b} \oplus \neg c)$

a	b	c	$\neg \boldsymbol{b}$	$a \to \neg b$	$\neg \boldsymbol{c}$	$\boldsymbol{b} \oplus \neg \boldsymbol{c}$	Result
T	T	T	F	F	F	T	F
T	T	F	F	F	T	F	F
T	F	T	T	T	F	F	F
T	F	F	T	T	T	T	T
F	T	T	F	T	F	T	T
F	T	F	F	T	T	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	T	T	T

2. Translate the English statement into propositional logic - *"If I get paid today, I will not go to Spain and London."*

p: I get paid today. q: I will go to Spain. r: I will go to London.	$p \to \neg(q \land r)$

3. Negate the English statement given in question 2

$$\neg [p \to \neg(q \land r)] \equiv [p \land (q \land r)]$$

I get paid today, And I will go to Spain And London.

4. Write the converse, inverse and contrapositive of the propositional logic obtained from question 2.

Contrapositive (English and logic)	If I will go to Spain And London, then I did not get paid today.
	$\neg\neg(q \land r) \to \neg p \equiv (q \land r) \to \neg p$
Inverse (English and logic)	If I did not get paid today, then I will go to Spain And London.
	$\neg p \to \neg\neg(q \land r) \equiv \neg p \to (q \land r)$
Converse (English and logic)	If I will not go to Spain And London, then I get paid today
	$\neg(q \land r) \to p$

5. Show that $\neg(p \vee (\neg p \wedge q)) \equiv (\neg p \wedge \neg q)$ using truth tables

p	**q**	**¬p**	**¬q**	$\neg(\boldsymbol{p} \vee (\neg \boldsymbol{p} \wedge \boldsymbol{q}))$	$(\neg \boldsymbol{p} \wedge \neg \boldsymbol{q})$
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

6. For Questions 6a and 6b, translate the English statements into predicate logic where the domain of students is all students in FCIT
   Hint: Use S(x) – x is a student in FCIT and J(x) – x lives in Jeddah

6a: "Some students *in FCIT lives in Jeddah"*

$\exists x\, J(x)$ and $\exists x(S(x) \wedge J(x))$

6b: "All students in FCIT lives in Jeddah"

$\forall x\, J(x)$ and $\forall x(S(x) \to J(x))$

7. Show that $[\neg q \wedge (p \to q)] \to \neg p$ is a tautology

Expression	**Rule**
$[\neg q \wedge (p \to q)] \to \neg p$	Implication Law.
$[\neg q \wedge (\neg p \vee q)] \to \neg p$	Distributive Law.
$[(\neg q \wedge \neg p) \vee (\neg q \wedge q)] \to \neg p$	Negation Law.
$[(\neg q \wedge \neg p) \vee (\text{False})] \to \neg p$	Identity Law.
$(\neg q \wedge \neg p) \to \neg p$	Implication Law.
$\neg(\neg q \wedge \neg p) \vee \neg p$	De Morgan Law.
$(\neg\neg q \vee \neg\neg p) \vee \neg p$	Double Negation Law.
$(q \vee p) \vee \neg p$	Associative Law.
$q \vee (p \vee \neg p)$	Negation Law.
$q \vee \text{True}$	Domination Law.
True	

8. Given below are arguments and a rule of inference. Write the correct rule in each case.

"If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn."

p: I go swimming. q: I will stay in the sun too long. r: I will sunburn.	$p \to q$ $\underline{q \to r}$ $\therefore p \to r$ Hypothetical Syllogism rule.

9. Each row in the following table contains independent premises. Show what can be concluded, if it is possible, and give the name of the rule in each.

1.	$\forall x(P(x) \to Q(x))$ ________________ $\therefore P(a) \to Q(a)$	Universal Instantiation
2.	$P(a) \land \neg R(a)$ ________________ $\therefore \exists x\,(P(x) \land \neg R(x))$	Existential Generalization
3.	$p \to s$ s ________________ ∴ not possible	not possible
4.	$p \to s$ $\neg p$ ________________ ∴ not possible	not possible
5.	$\neg p \to s$ $\neg p$ ________________ $\therefore s$	Modus Ponens

10. If m is an odd number and n is an even number, prove that the expression $m^2 + mn - 1$ is even.
p: m is an odd number and n is an even number.
q: $m^2 + mn - 1$ is even.
<u>Direct Proof: Assume that p is true. Use rules of inference, axioms, and logical equivalences to show that q must also be true</u>.
Assume that m is odd. Then n = 2k + 1 for an integer k.
Assume that n is even. Then n = 2k for an integer k.
$m^2 + mn - 1 = (2k + 1)^2 + (2k + 1)\times(2k) - 1 = (4k^2 + 4k + 1^2) + (4k^2 + 2k) - 1 = 8k^2 + 6k + 1 - 1 = 8k^2 + 6k = 2(4k^2 + 3k) = 2r$, where $r = (4k^2 + 3k)$, an integer.
We have proved that If m is an odd number and n is an even number, then $m^2 + mn - 1$ is even.

- QED.