**Lab 10 Relations**

**Objective**

Solving exercises from the text book.

**Current Lab Learning Outcomes (LLO)**

By completion of the lab, the students should be able to

1. Understand Relations and representation of relation.

2. Understand the properties of a relation

3. Solve shorter/easier or longer / harder problems given in the textbook.

**Lab Requirements**

Students allowed using their lecture notes in the lab and use blackboard slides in order to solve the exercises.

**Lab Assessment**

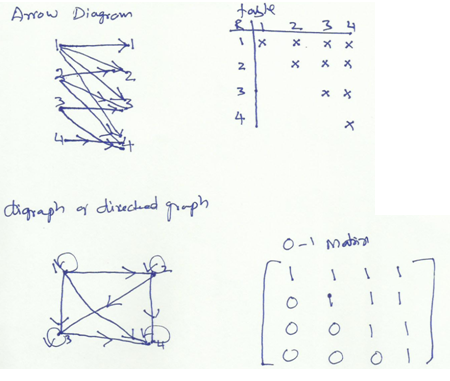
1- Divide students to groups and let them to solve the given example.

2- Discuss the answers with the groups and write on board the optimal solution

1. List the ordered pairs in the relation *R* from *A* = {0*,* 1*,* 2*,* 3*,* 4} to *B* = {0*,* 1*,* 2*,* 3}, where *(a, b)* ∈ *R* if and only if

**a)** *a* = *b*. **b)** *a* + *b* = 4. **c)** *a > b*. **d)** *a* | *b*. **e)** gcd*(a, b)* = 1. **f )** lcm*(a, b)* = 2.

1. {*(*0*,* 0*), (*1*,* 1*), (*2*,* 2*), (*3*,* 3*)*} **b)** {*(*1*,* 3*), (*2*,* 2*),(*3*,* 1*), (*4*,* 0*)*} **c)** {*(*1*,* 0*), (*2*,* 0*), (*2*,* 1*), (*3*,* 0*), (*3*,* 1*), (*3*,* 2*),(*4*,* 0*), (*4*,* 1*), (*4*,* 2*), (*4*,* 3*)*} **d)** {*(*1*,* 0*), (*1*,* 1*), (*1*,* 2*), (*1*,* 3*),(*2*,* 0*), (*2*,* 2*), (*3*,* 0*), (*3*,* 3*), (*4*,* 0*)*} **e)** {*(*0*,* 1*), (*1*,* 0*), (*1*,* 1*),(*1*,* 2*), (*1*,* 3*), (*2*,* 1*), (*2*,* 3*), (*3*,* 1*), (*3*,* 2*), (*4*,* 1*), (*4*,* 3*)*}**f)** {*(*1*,* 2*), (*2*,* 1*), (*2*,* 2*)*}
2. Represent the relation R={(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3) (3,4),(4,4)} as Arrow diagram, digraph, table and as a 0-1 matrix.



1. For each of these relations on the set {1*,* 2*,* 3*,* 4}, determine whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

**a)** {*(*2*,* 2*), (*2*,* 3*), (*2*,* 4*), (*3*,* 2*), (*3*,* 3*), (*3*,* 4*)*}

**b)** {*(*1*,* 1*), (*1*,* 2*), (*2*,* 1*), (*2*,* 2*), (*3*,* 3*), (*4*,* 4*)*}

**c)** {*(*2*,* 4*), (*4*,* 2*)*}

**d)** {*(*1*,* 2*), (*2*,* 3*), (*3*,* 4*)*}

**e)** {*(*1*,* 1*), (*2*,* 2*), (*3*,* 3*), (*4*,* 4*)*}

Solution: **a)** Transitive **b)** Reflexive,symmetric, transitive **c)** Symmetric **d)** Antisymmetric**e)** Reflexive, Symmetric, Transitive,Anti Symmetric

1. Give an example of an irreflexive relation on the set of all people.

Short man and tall man

1. Let *R*1 = {*(*1*,* 2*), (*2*,* 3*), (*3*,* 4*)*} and *R*2 = {*(*1*,* 1*), (*1*,* 2*)*, *(*2*,* 1*)*, *(*2*,* 2*)*, *(*2*,* 3*)*, *(*3*,* 1*)*, *(*3*,* 2*)*, *(*3*,* 3*)*, *(*3*,* 4*)*} be relationsfrom {1*,* 2*,* 3} to {1*,* 2*,* 3*,* 4}. Find

**a)** *R*1 ∪ *R*2. **b)** *R*1 ∩ *R*2. **c)** *R*1 − *R*2. **d)** *R*2 − *R*1.

a) {*(*1*,* 1*), (*1*,* 2*)*, *(*2*,* 1*)*, *(*2*,* 2*)*, *(*2*,* 3*)*, *(*3*,* 1*)*, *(*3*,* 2*)*, *(*3*,* 3*)*, *(*3*,* 4*)*} b) {*(*1*,* 2*), (*2*,* 3*), (*3*,* 4*)*}

c) {} d) {*(*1*,* 1*)*, *(*2*,* 1*)*, *(*2*,* 2*)*, *(*3*,* 1*)*, *(*3*,* 2*)*, *(*3*,* 3*)*}

1. Determine whether the relation *R* on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where *(x, y)* ∈ *R* if and only if

**a)** *x* + *y* = 0. **b)** *x* = ±*y*. **c)** *x* − *y* is a rational number.

a. not reflexive ( 1+1 ≠ 0), symmetric (x+y=y+x=0) not anti symmetric , not transitive

b. reflexive (x=±*x) x=*±*y* means x ∈ {−y, y} , symmetric (*x* = ±*y*. neans *y*= ±*x) transitive , x* = ±*y*.

, *y* = ±*z*. *z*= ±*x*. c) reflexive (x-x=0, 0 is rational) symmetric (x-y is rational, y-x is also rational)not symmetric (1 – 2 rational, 2 -1 rational and 2 ≠ 1), transitive 9x-y is rational, y-x is rational and z-x is also rational)

1. Let *R* be the relation on the set {0*,* 1*,* 2*,* 3} containing the ordered pairs *(*0*,* 1*)*, *(*1*,* 1*)*, *(*1*,* 2*)*, *(*2*,* 0*)*, *(*2*,* 2*)*, and *(*3*,* 0*)*. Find the
   1. reflexive closure of *R*. **b)** symmetric closure of *R*.

**Solution a)** {*(*0*,* 0*)*, *(*0*,* 1*)*, *(*1*,* 1*)*, *(*1*,* 2*)*, *(*2*,* 0*)*, *(*2*,* 2*)*, *(*3*,* 0*)*, *(*3*,* 3*)*}**b)** {*(*0*,* 1*)*, *(*0*,* 2*)*, *(*0*,* 3*)*, *(*1*,* 0*)*, *(*1*,* 1*)*, *(*1*,* 2*)*, *(*2*,* 0*)*, *(*2*,* 1*)*,*(*2*,* 2*)*, *(*3*,* 0*)*}

1. Which of these relations on {0*,* 1*,* 2*,* 3} are equivalence relations? Determine the properties of an equivalence relation that the others lack.

**a)**{*(*0*,* 0*), (*1*,* 1*), (*1*,* 3*), (*2*,* 2*), (*2*,* 3*), (*3*,* 1*), (*3*,* 2*),(*3*,* 3*)*} **b)** {*(*0*,* 0*), (*0*,* 1*), (*0*,* 2*), (*1*,* 0*), (*1*,* 1*), (*1*,* 2*), (*2*,* 0*),(*2*,* 2*), (*3*,* 3*)*}

Solution: **a)** Not transitive **b)** Not symmetric, not transitive

1. What is the composite of the relations *R* and *S*, where *R* is the relation from {1*,* 2*,* 3} to {1*,* 2*,* 3*,* 4} with *R* = {*(*1*,* 1*), (*1*,* 4*), (*2*,* 3*), (*3*,* 1*), (*3*,* 4*)*} and *S* is the relation from {1*,* 2*,* 3*,* 4} to {0*,* 1*,* 2} with *S* = {*(*1*,* 0*), (*2*,* 0*), (*3*,* 1*), (*3*,* 2*), (*4*,* 1*)*}?

*Solution: S* ◦*R* is constructed using all ordered pairs in *R* and ordered pairs in *S*, where the second element of the ordered pair in *R* agrees with the first element of the ordered pair in *S*. For example, the ordered pairs *(*2*,* 3*)* in *R* and *(*3*,* 1*)* in *S* produce the ordered pair *(*2*,* 1*)* in *S* ◦*R*. Computing all the ordered pairs in the composite, we find

*S* ◦*R* = {*(*1*,* 0*), (*1*,* 1*), (*2*,* 1*), (*2*,* 2*), (*3*,* 0*), (*3*,* 1*)*}*.*

1. Let *R* be the relation on the set {1*,* 2*,* 3*,* 4*,* 5} containing the ordered pairs *(*1*,* 1*)*, *(*1*,* 2*)*, *(*1*,* 3*)*, *(*2*,* 3*)*, *(*2*,* 4*)*, *(*3*,* 1*)*,*(*3*,* 4*)*, *(*3*,* 5*)*, *(*4*,* 2*)*, *(*4*,* 5*)*, *(*5*,* 1*)*, *(*5*,* 2*)*, and *(*5*,* 4*)*. Find

**a)** *R*2. **b)** *R*3.

**a.** R2=R◦R ={(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,4),(2,5),(2,2),(3,1),(3,2),(3,3),(3,5),(3,4),(4,3),(4,4),(4,1),(4,2),(5,1),(5,2),(5,3),(5,4),(5,5)}

**b.** *R*3= *R*2◦*R* ={(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,4),(2,3)(2,5),(2,2),(3,1),(3,2),(3,3),(3,5),(3,4),(4,3),(4,4),(4,5),(4,1),(4,2),(5,1),(5,2),(5,3),(5,4),(5,5)}

1. Find the transitive closures of these relations on {1*,* 2*,* 3*,* 4}.

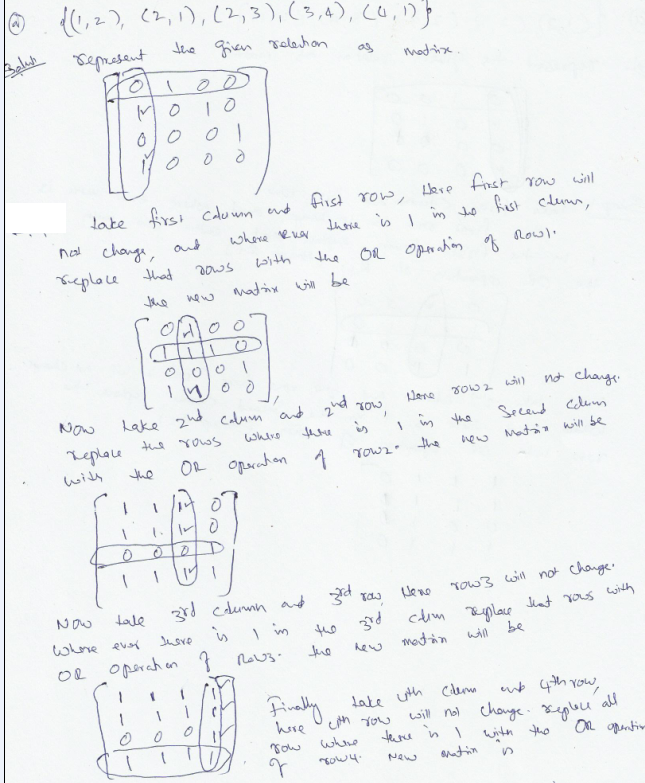
**a)** {*(*1*,* 2*), (*2*,*1*), (*2*,*3*), (*3*,*4*), (*4*,*1*)*} **b)** {*(*2*,* 1*), (*2*,*3*), (*3*,*1*), (*3*,*4*), (*4*,*1*), (*4*,* 3*)*} **c)** {*(*1*,* 2*), (*1*,*3*), (*1*,*4*), (*2*,*3*), (*2*,*4*), (*3*,* 4*)*} **d)** {*(*1*,* 1*), (*1*,*4*), (*2*,*1*), (*2*,*3*), (*3*,*1*), (*3*,* 2*), (*3*,*4*), (*4*,* 2*)*}

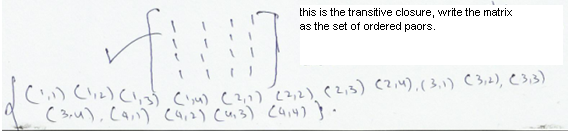


Method:

1. First represent the relation in the matrix form.
2. Take the first column and first row, where ever there exists 1 in the column , replace those rows with the OR operation of Row 1.(row 1 remains unchanged).resulting in a new matrix.
3. Now take second column and second row, where ever there exist 1 in the second column, replace the entire row with OR operation of row 1.(row 2 will be unchanged),
4. Now take third column and third row, where ever there exists 1 in the third column, replace the row with OR operator of row 3.(here row 3 is unchanged).
5. Finally take the fourth column and fourth row, where ever there exists 1 in the 4th column, replace the row with OR operation of row4 (here row 4 is unchanged). This new matrix is the transitive closure of the relation. Write all the ordered pairs from this matrix.

Here is the explanation for first question.





The answers for question b,c,d are given below in the form of matrix.

