**Lab 04 Predicates and Quantifiers**

**Objective**

Solving exercises from the textbook in chapter 1.4-1.5 (partial from 1.6)

**Current Lab Learning Outcomes (LLO)**

By completion of the lab, the students should be able to:

1. Will be able to identify the truth values of the quantifiers and nested quantifiers, negate the quantifiers.

2. They will be able to solve shorter/easier or longer / harder problems given in the textbook.

**Lab Requirements**

Students allowed using their lecture notes in the lab in order to solve the exercises.

**Lab Assessment**

1- Divide students to groups and let them to solve the given example.

2- Discuss the answers with the groups and write on board the optimal solution.

**Lab Description**

1. Let *N(x)* be the statement “*x* has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.
   1. ∃*xN(x)* **b)** ∀*xN(x)* **c)** ￢∃*xN(x)***d)** ∃*x*￢*N(x)* **e)** ￢∀*xN(x)* **f )** ∀*x*￢*N(x)*

Solution:

a) ∃*xN(x)*

1. there are some students who has visited north Dakota.

**b)** ∀*xN(x)*

b. all the students who has visited north Dakota

**c)** ￢∃*xN(x)*

c. all the students not visited north Dakota .

or no student in the school has visited North Dakota.

(Alternatively, there does not exist a student in the school who has visited North Dakota.)

**d)** ∃*x*￢*N(x)*

d. there are some students who has not visited north Dakota,

**e)** ￢∀*xN(x)*

e. It is not true that every student in the school has visited North Dakota. (= d)

(Alternatively, not all students in the school have visited North Dakota.)

or there are some students who has not visited north Dakota

**f )** ∀*x*￢*N(x)*

f. all the students not visited north Dakota (= c)

or there is no student who has visited north Dakota

1. Translate these statements into English, where *C(x)* is “*x* is a comedian” and *F(x)* is “*x* is funny” and the domain consists of all people.

**a)** ∀*x(C(x)* → *F(x))* **b)** ∀*x(C(x)* ∧ *F(x))***c)** ∃*x(C(x)* → *F(x))* **d)** ∃*x(C(x)* ∧ *F(x))*

Solution:

**a)** Every comedian is funny.

**b)** Every person is a funny comedian.

**c)** There exists a person such that if she or he is a comedian, then she or he is funny.

**d)** Some comedians are funny.

1. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
   1. Someone in your class can speak Hindi.
   2. Everyone in your class is friendly.
   3. There is a person in your class who was not born in California.
   4. A student in your class has been in a movie.
   5. No student in your class has taken a course in logic programming.

**Let *C(x)* be the propositional function “*x* is in your class.”**

**a) ∃*xH(x)* and ∃*x(C(x)* ∧*H(x))*, where *H(x)* is “*x* can speak Hindi”**

**b) ∀*xF(x)* and ∀*x(C(x)* →*F(x))*, where *F(x)* is “*x* is friendly”**

**c) ∃*x*￢*B(x)* and ∃*x(C(x)*∧￢*B(x))*, where *B(x)* is“*x* was born in California”**

**d) ∃*xM(x)* and ∃*x(C(x)*∧*M(x))*,where *M(x)* is “*x* has been in a movie”**

**e) ∀*x*￢*L(x)* and ∀*x(C(x)* →￢*L(x))*, where *L(x)* is “*x* has taken a course in logic programming”**

1. Determine the truth value of each of these statements if the domain consists of all real numbers.

**a)** ∃*x(x*3 = −1*)* **b)** ∃*x(x*4 *< x*2*)***c)** ∀*x((*−*x)*2 = *x*2*)* **d)** ∀*x(*2*x > x)*

Solution: a is true , because if x=-1 then x3=-1 b. True , because if x=1/2, then x2=1/4 and x4=1/16, so *x*4 *< x*2  c true d. false [0, - inf]

1. Determine the truth value of each of these statements if the domain consists of all integers.

**a)** ∀*n(n*2 ≥ 0*)* **b)** ∃*n(n*2 = 2*)***c)** ∀*n(n*2 ≥ *n)* **d)** ∃*n(n*2 *<* 0*)*

a)True b)False c)True d)False

1. Let S(x) be the predicate “x is a student,” F(x) the predicate “x is a faculty member,” and A(x, y) the predicate “x has asked y a question,” where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.
2. Lois has asked Professor Michaels a question.
3. Every student has asked Professor Gross a question.
4. Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.
5. Some student has not asked any faculty member a question.

a) A(Lois, Professor Michaels)

b) ∀x(S(x) → A(x, Professor Gross))

c) ∀x(F(x) → (A(x,Professor Miller) ∨ A(Professor Miller, x)))

d) ∃x(S(x) ∧∀y(F(y) →￢A(x, y)))

1. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

**a)** ∀*n*∃*m(n*2 *< m)* **=True**

**b)** ∃*n*∀*m(n < m*2*)***=True**

**c)** ∀*n*∃*m(n* + *m* = 0*)* **=True**

**d)** ∃*n*∀*m(nm* = *m)***=True ; n == 1**

1. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

**a)** ∀*x*∃*yP(x, y)* ∨∀*x*∃*yQ(x, y)*

**b)** ∀*x*∃*y(P(x, y)* ∧∃*zR(x, y, z))*

**c)** ∀*x*∃*y(P(x, y)* →*Q(x, y))*

**a)** ∃*x*∀*y*￢*P(x, y)* ∧∃*x*∀*y* ￢ *Q(x, y)*

**b** ∃*x*∀*y (*￢*P(x, y)*∨∀*z*￢*R(x, y, z))*

**c)** ∃*x*∀*y(P(x, y)*∧￢*Q(x, y))*

1. Express the negation of these propositions using quantifiers, and then express the negation in English.

**a)** Some drivers do not obey the speed limit.

**b)** All Swedish movies are serious.

**c)** No one can keep a secret.

**d)** There is someone in this class who does not have a good attitude.

a. Some drivers do not obey the speed limit. Let S(x) be “x obeys the speed limit,” where the domain is drivers. The original statement is ∃x¬ S(x), the negation is ∀xS(x), “All drivers obey the speed limit.”

b. All Swedish movies are serious. Let S(x) be “x is serious,” where the domain is Swedish movies. The original statement is ∀xS(x), the negation is ∃x¬S(x), “Some Swedish movies are not serious.”

c. No one can keep a secret. Let S(x) be “x can keep a secret,” where the domain is people. The original statement is ¬∃xS(x), the negation is ∃xS(x), “Some people can keep a secret.”

d. There is someone in this class who does not have a good attitude. Let A(x) be “x has a good attitude,” where the domain of discourse is people in this class. The original statement is ∃x¬A(x), the negation is ∀xA(x), “Everyone in this class has a good attitude

1. Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

**a)** ∃*x*∀*y(xy* = *y)*

**b)** ∀*x*∀*y(((x <* 0*)* ∧ *(y <* 0*))* → *(xy >* 0*))*

**c)** ∃*x*∃*y((x*2 *> y)* ∧ *(x < y))*

**d)** ∀*x*∀*y*∃*z(x* + *y* = *z*

**a)** There is a multiplicative identity for the real numbers. or for any real number y there is at least one value x such that the product of x and y is y

**b)** The product of two negative real numbers is always a positive real number.

**c)** There exist real numbers *x* and *y* such that *x*2 exceeds *y* but *x* is less than *y*.

**d)** The real numbers are closed under the operation of addition. or addition of two real numbers is a real number

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If George does not have eight legs, then he is not a spider. George is a spider.

∴ George has eight legs

p :“George does not have eight legs” q : “George is not a spider.” First statement: p → q The second statement is ¬q Using modus tollens. The conclusion is: ¬ p

1. What rule of inference is used in each of these arguments?
2. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

Modus tollens

**b)** If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn

Hypothetical syllogism.

1. For each of these arguments determine whether the argument is correct or incorrect and explain why.

**a)** All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

**Hw b)** Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.

**Hw c)** All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

**d)** Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

Solution: **a)** Correct, using universal instantiation and modus ponens **b)** Invalid; fallacy of affirming the conclusion **c)** Invalid; fallacy of denying the hypothesis **d)** Correct, using universal instantiation and modus tollens





